

Incorporating Interactive Mathematical Content into Web-Based Distance-Learning Systems

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Abstract

The quality of digital courseware and books available on the web is evolving rapidly, much however remains to be done! As matters stand, electronic educational documentation is still in its infancy. In the mathematical domain, the lack of interactivity and the rather poor quality of search engines make educational courseware and books even less attractive. This is due to the fact that viewing and manipulating mathematical objects is much more difficult than working with pure text documents.

We present in this document the tools and techniques we have developed and are currently developing as partners in two different European projects: the OpenMath project which ended recently (Esprit project 24969 "Accessing and Using Mathematical Information Electronically" which led to the creation of the OpenMath standard for describing mathematical objects) and TRIAL-SOLUTION, IST project 11397 which started a year ago. These tools deal with the editing of mathematical content and books, structured using metadata and slicing technologies.

Keywords

Distance learning, mathematical interactive documents, Java Beans, OpenMath, XML, Metadata

Introduction

More reusability and more flexibility

Today, most educational electronic documents are still very similar to paper documents – most authors still just publish their files on the web, with very few enhancements. They use their own tools (computer, word processor, web authoring tools, etc.) for producing their texts, slides, web pages, independently from their colleagues. Exchanging these documents is often difficult, even from/to the same version of software that has produced the documents if it runs on different platforms. ASCII text is easy to exchange but in the scientific fields, formulæ and mathematical content have no real standard (except for TeX/LaTeX users). Even when exchange of documents is possible, it remains very difficult to find and exploit their content several months after their publication on the web, because search engines rarely index them correctly. The reason is *the lack of semantic information*.

Even worse, the content is almost always closely tied to its presentation and several versions of the same content may co-exist: as slides, as a report, etc. Recent approaches based on technologies such as metadata and XML propose interesting solutions to these problems.

Within the TRIAL-SOLUTION project (Tools for Reusable Integrated Adaptable Learning - Systems/standards for Open Learning Using Tested Interoperable Objects and Networking), slicing, authoring and delivery tools are being developed for the creation and exploitation of metadata-enhanced documents on the web. Slicing and authoring tools provide means for slicing and adding metadata to source documents without worrying about their presentation. Delivery tools provide a front end, a web-

based interface for publishing and exploiting interactive mathematical courseware produced by the authoring tool. Current prototype versions of these tools are presented in further sections.

Metadata enhance the semantic value of digital documents by adding hidden information such as “this chapter talks about 3D surfaces”, and “this is a subtopic of the Geometry topic” or “Chapter 3 has to be read before you study the example given in this paragraph”. Contextual information such as the topic of a given part of a document, prerequisites, author name and distribution policy, etc. are metadata. Standards for metadata representation and description have recently started to emerge.

The well-known "Dublin Core Metadata Document Set", developed by an international, interdisciplinary commission has become a de facto standard [6]. It defines three sets of metadata that defines the resource content such as "Title" or "Subject", elements associated to the intellectual property of a resource, such as "Author" or "Editor" and finally elements associated to the instance of a resource, like "Date" or "Format". Another standard, Instructional Management Systems (IMS [8]) extends Dublin Core by adding elements for describing educational content.

How to implement a metadata set is an important question. First, it is good to rely on a worldwide standard if one plans to protect one's investment. The second fundamental idea is to separate the document content from its presentation. XML [13] XHTML, <http://www.w3.org/TR/xhtml1/>

[14] seems to be the most suitable language for describing both the structure of a document and the associated metadata. Definitions of these concepts, as well as examples illustrated by MAT, a tool we are developing the Trial Solution project, are given below to illustrate this new approach.

More interactivity in mathematical courseware on the web

The presentation of mathematics on the World Wide Web currently leaves much to be desired. Indeed, only a few ways of rendering mathematical formulæ exist. The most common is to embed bitmaps into HTML documents. However, the main drawbacks with this solution are that there is no interactivity, and it is impossible to extract the mathematical notion displayed. Clearly, a new way of representing mathematical information is needed.

The OpenMath European project gave birth to a standard for a formal description of mathematical objects, enabling their electronic exchange and manipulation. Whereas MathML [10] pays more attention to the graphical representation of mathematical expressions, OpenMath describes the semantics of the objects, allowing unambiguous communication between tools such as computer algebra systems, databases, graphing tools, etc.

These technologies and these tools naturally find application in the field of distance learning. Several examples articulated around JOME, a tool we have developed, are presented in this document.

MAT: Mainline Authoring Tool

This authoring tool allows the user to control document slicing, XML view-independant representation of the document structure, metadata edition, thesaurus edition, etc. It is still in its early stages of development.

This tool has an open architecture designed to be extensible in order to support a large palette of input formats. We just suppose that the input document is described using an XML syntax. XML is a worldwide standard, XML documents are easy to parse, manipulate and can be translated easily into many output formats. Much has already been published about XML, see for example <http://www.xml.org/>.

Furthermore, many tools already use an XML syntax for describing the documents they produce, e.g., Office 2000 or Framemaker. Translators are already available or under development for many other common formats like HTML or LaTeX. We can reasonably expect that translators from any input format to XML will be available quite soon.

For the moment, our tool only supports XHTML documents [13] as input. XHTML is an XML syntax of HTML. Such documents can be obtained very easily using a tool like Tidy [12], that translates HTML documents, even corrupted, into clean, easy-to-parse XHTML.

For us, supporting a new XML syntax, e.g., as used by Word 2000 documents, will involve a new input plugin, which should be quite easy to write, thanks to the open architecture of our tool.

Finding out the document structure is straightforward once the XML tags related to the structural part of the document have been identified. In the case of XHTML, the well-known HTML tags H1, H2, H3 belong to this category.

Figure 1 shows a picture of MAT after an XHTML document has been opened (from a course about the Swing Java package). On the left, the structured tree of the document is displayed and on the right we can see the view corresponding to the selected part. The view is the graphic rendering of the selected part. That mode makes selection of parts of the document intuitive. The user can select a whole chapter or a paragraph by clicking directly in the graphic tree, or can select only a few words or sentences by selecting parts of the displayed document on the right, directly in the view area. Once a selection has been performed, metadata associated to the selected part of the document can be edited.

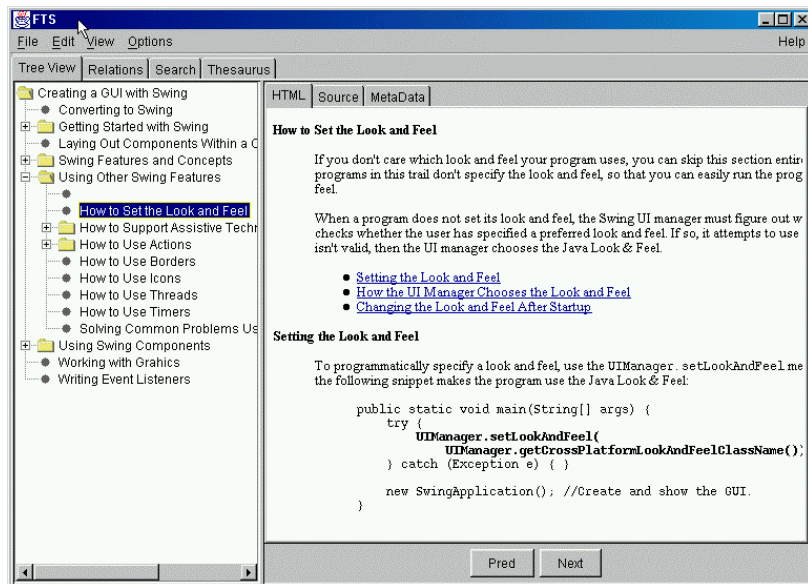
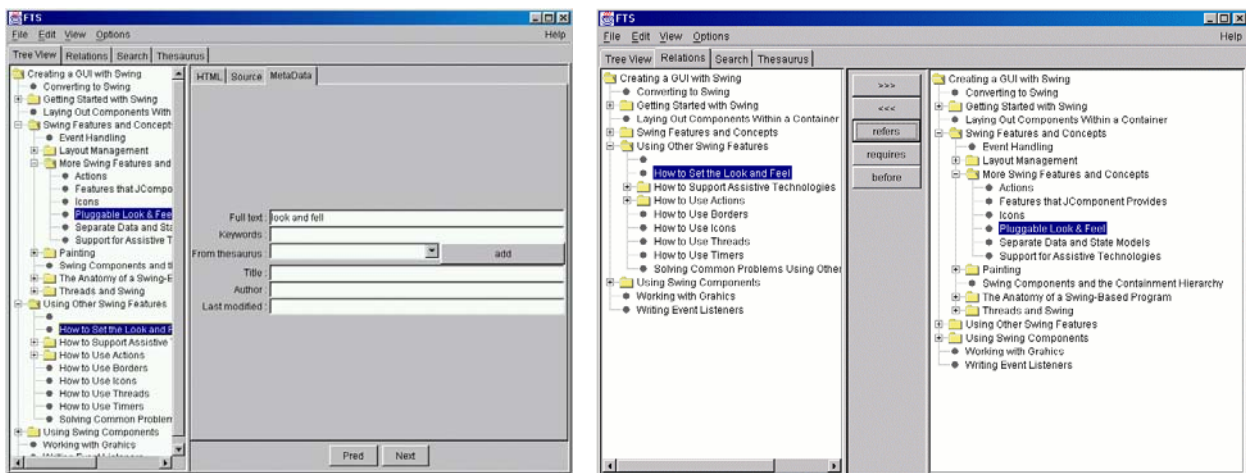


Figure 1 : On the left the structure tree of the document, on the right the view of the selected part.



Editing of the metadata associated to the part of the document selected in Figure 1

The screen used to define relationships between parts of the document

Figure 2: Metadata edition

Figure 2 shows how metadata can be added to the selected part of the document, and how relationships can be defined.

An important feature of the tool is that with the help of a thesaurus specialized for the domain of interest, it can automatically add metadata to parts of the document using other tools developed by the project members. For example, by localizing where particular sentences or groups of words appear, one can guess the topic of a paragraph or a chapter. This can provide hints to the user of the authoring tool that will help him save time with metadata editing.

However, when dealing with formulæ, a simple text-based thesaurus is not enough. A standard formal description for mathematical objects is required, hence the development of OpenMath and JOME!

OpenMath

Communication is a key point in the elaboration of a distance-learning system, and because mathematical communication must be clear and unambiguous, a formal language is required to describe mathematical notations. OpenMath [4] acts as such. It is an XML application and represents a platform independent standard for a semantically rich encoding of mathematical concepts. JOME, our OpenMath editor, (see below), is OpenMath compliant.

Meaning is assigned to OpenMath objects through XML documents called Content Dictionaries. These documents hold the definition (i.e. the semantics) of each symbol appearing in OpenMath objects. For example, the multiplication will have different properties whether referring to matrices (multiplication is not commutative) or integers (multiplication is commutative). This means we have two different operators, each one being defined in a different Content Dictionary. Thus, a computer algebra system will know what kind of manipulations are allowed in a given computation.

The standard provides a set of official Content Dictionaries which are adopted and approved by the OpenMath Society [11]. Each of these Content Dictionaries covers a specific mathematical domain, such as arithmetic, algebra, statistics, etc.

OpenMath provides two different kinds of encoding: an XML encoding, which is the standard encoding, and a binary encoding when large amount of data have to be exchanged between applications. The OpenMath encoding of $\sin(x)$ is presented Figure 3.

```
<OMOBJ>
  <OMA>
    <OMS cd=transc1" name="sin"/>
    <OMV name="x"/>
  </OMA>
</OMOBJ>
```

Figure 3: OpenMath encoding of $\sin(x)$ formula

This encoding says that the symbol \sin tagged by OMS is defined by the transc1 Content Dictionary. The element OMA and OMV identify respectively applications and variable elements.

OpenMath is based on a three layer architecture:

1. a private layer which corresponds to the internal representation used by the application,
2. an abstract layer which is the OpenMath representation of the private layer,
3. a communication layer which transforms the OpenMath object into a byte stream.

Thus, a program A internally manipulates mathematical data using its own data structures, transforms them into OpenMath objects which it sends to another program B, using the communication layer. In its turn, program B converts the OpenMath objects received into its own data structure and does the appropriate treatment. That way, heterogeneous applications can easily communicate mathematical objects.

JOME: Java OpenMath Editor

Distance learning of mathematics over the World Wide Web is currently not widespread due to a lack of generic tools. Indeed, representing and manipulating mathematical expressions, which need a two-dimensional notation, special symbols, etc. requires the implementation of quite complex techniques. In order to incorporate interactive mathematical content into web-based distance learning systems, we have developed JOME [5][2], a self-contained software component which allows editing, visualization, exportation and importation of OpenMath objects. Designed as a Java Bean [9], the Java standard for software components, it can easily be integrated inside complex applications where the manipulation of mathematical formulæ is required.

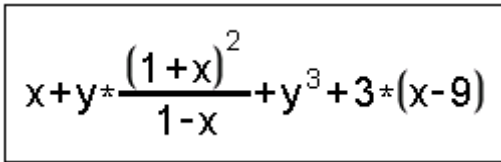

$$x + y * \frac{(1+x)^2}{1-x} + y^3 + 3*(x-9)$$

Figure 4: JOME displaying a formula

Software Component: Java Bean

Software components are to software developers what Lego is to children. They simplify the design of complex applications, in principle without writing a single line of code, thanks to visual application builder tools such as IBM Visual Age, Symantec Visual Café, etc.

Java builder tools maintain Java Beans in a palette or toolbox. At design-time, i.e., for integration into a larger component, one can select a particular Java Bean from the toolbox, drop it into a workspace, modify its appearance and behavior, define its interaction with other Java Beans, etc. All this is carried out mostly with the mouse, hence open to non-programmers.

Interactivity

Nowadays, just displaying a formula is not enough: interaction is essential. JOME proposes the following mechanisms for interacting with the displayed formula:

- **selection/deselection**

Selection is the first brick of functionality for interactivity. It determines the part of the formula to which the desired manipulations will apply. JOME automatically fits the selected sub-expression to be the smallest syntactically correct expression according to the element pointed to by the mouse (and modified by other selections). This guarantees the coherence of subsequent operations on the expression. It is possible to select the whole formula or just parts of it. Figure 5 illustrates the selection of a sub-expression of the formula displayed in Figure 4.

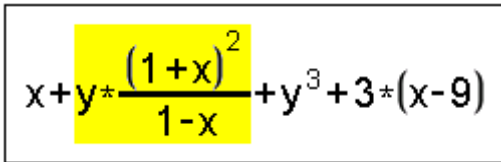

$$x + y * \frac{(1+x)^2}{1-x} + y^3 + 3*(x-9)$$

Figure 5: Selection of a sub-expression in JOME

- **semantic drag and drop**

Once selected, the sub-expression can be manipulated according to the properties of the applicable operator. For example, taking into account the commutativity property of addition, it is possible to "move" the selected operands. If a and b were matrices, this would of course not have been allowed

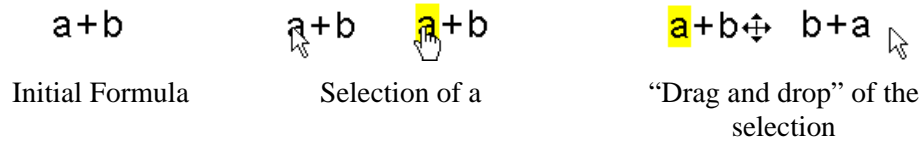


Figure 6: “Drag and drop” of s selection

Figure 6 shows the result of a drag and drop of the selection. Further development of this functionality might consider the properties of an operator over other operators. For example, distributivity of multiplication over addition, equality laws, etc. This allows the user to make basic algebraic manipulations [1].

- **iconification/de-iconification**

This allows the selection to be replaced by an icon. The icon reflects the part being iconified by taking the shape of a rectangle containing the symbol of the corresponding operation. For example, if the selection represents a multiplication, then the icon will represent a multiplication symbol (see Figure 7), etc.

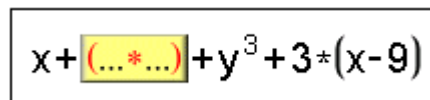


Figure 7: Iconification of a sub-expression in JOME

This iconification process can considerably reduce the space needed for the rendering of the formula. By carefully iconifying parts of a big formula, the space needed for its rendering can be considerably reduced while keeping it understandable (intelligent fold of the formula). The iconified part can also be unfolded in order to get more details.

JOME as the Core of a Web-Based Distance-Learning Systems

Most Web-based distance-learning systems use a classical client-server architecture (see Figure 8).

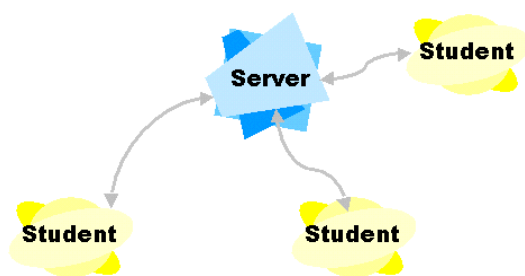


Figure 8: Classical client/server architecture

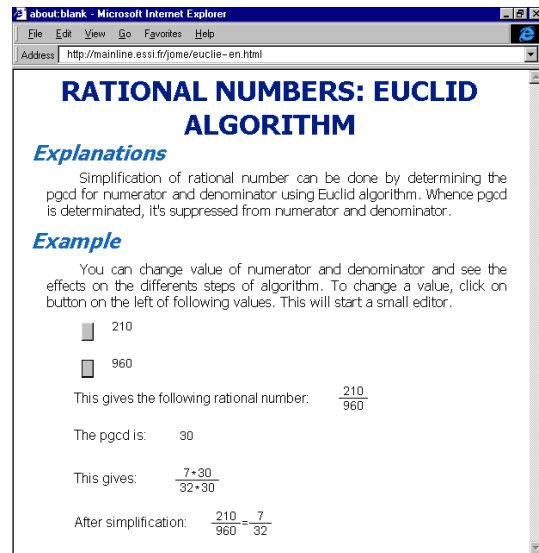


Figure 9: A JOME based interactive course

The server holds the course and provides, when requested, the HTML documents to the client. Incorporating JOME into such architecture can open the way to enhanced interactivity. As depicted in Figure 9, an interactive course dealing with the simplification of a rational number where each formula is displayed by the means of an applet using JOME, a change on the numerator (respectively the

denominator) is automatically forwarded to the dependencies to maintain coherence. What is exchanged between the applets are OpenMath objects.

It is important to consider the course presented Figure 9 as a first step in the design of interactive exercises. Indeed, specific applets have been created for particular purposes (e.g. PGCD calculation). A direct extension would be to consider polynomial fractions rather than rational numbers, as shown Figure 10. With the use of OpenMath, the necessary computation would be sent to a server-side computer algebra system.

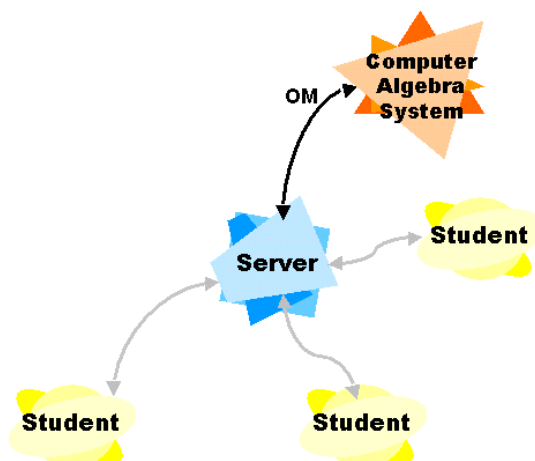


Figure 10: OpenMath server-side computer algebra system

When we think about interactive courses over the Web, certain questions arise: how to indicate his mistakes to a student, or how to give him help on a specific problem? Different methods already exist for such tasks, but are not always satisfactory. The most commonly used are forums. However, because of the format of the messages sent (i.e. simple text), it is frequently difficult to express mathematical information. For this issue, we have developed a talk application (Figure 12) which allows a teacher to communicate with students. The added value compared to existing applications is that mathematical information is no longer simple text but OpenMath objects. This brings a new dimension to our mathematics oriented distance-learning system: a human dimension (see Figure 11).

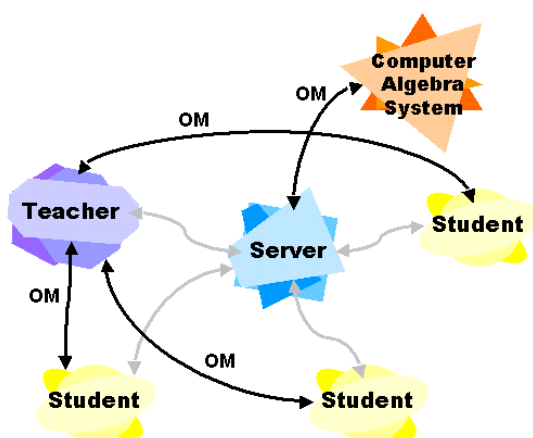


Figure 11: A third person to interact with the students: a teacher

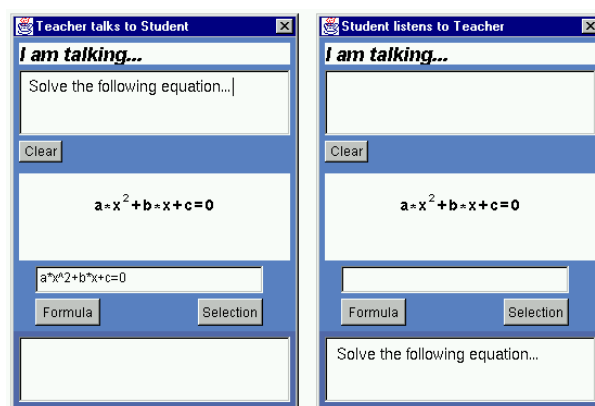


Figure 12: Mathematical talk between teacher and students

There is another important consideration about OpenMath to take into account: it not only represents an efficient mechanism for encoding the semantics of mathematical objects, it can also be used to define control commands for target applications. For example the Mfd2 [3] deductive database developed by the SAFIR project at INRIA. For that purpose, a new specific Content Dictionary has been created to define the request and response formats, and the different commands needed to control the

database. Once connected, a request typed in by the user is sent as an OpenMath object. The database deals with the request and sends back its answer as well as an OpenMath object (see Figure 13).

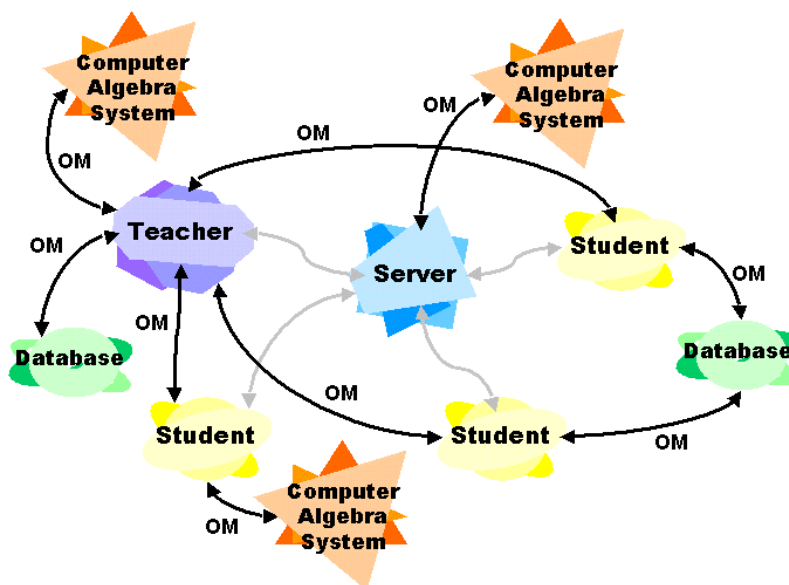


Figure 13: Web-based distance-learning environment for scientific course

This is an important feature because the user has the opportunity to find supplementary information, and more generally to enlarge his information source.

Conclusion

In this article, we have presented a complete platform design for interactive mathematical content for web-based learning systems. Examples cited in this article are a starting point for putting interactive mathematical content into web-based distant learning systems. Currently the OpenMath-based mathematical notation editor exists in the form of Java Bean software components. These can be used either as a standalone system or integrated into larger systems, useful for distance education for example. Our ongoing work in this direction will shortly lead to prototype implementations for experimental evaluations in the context of courses for engineer students.

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